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A Cross-Country Evidence

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Keywords: skill premium; factor-biased technological change; directed technological change

JEL classification: J24, J30, J31, O30, O31

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Factor-Biased Technological Change and the Skill Premium: A Cross-Country Evidence*

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Abstract

This paper provides a cross-country analysis of trends in the skill premium and in the relative supply of high-skilled labor based on the data for 21 OECD countries over the last decades of the twentieth century. We document that, in contrast to the steadily increasing trends in the relative supply of high-skilled labor, the dynamics of the skill premium varies substantially across the countries. One of the main empirical findings of the paper is the evidence that both US and European countries experienced skill-biased rather than unskill-biased technological change. We also develop a new modification of the Acemoglu’s directed technological change model and show that such modification is helpful in explaining the skill premium patterns.


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1 Introduction

The last decades are accompanied by significant changes in the US and European labor markets, which consequently affect the distribution of wages across skills. In particular, in the US a large increase in the skill premium started in the 1980s (see e.g. Acemoglu 2002b; Goldin and Katz 2008). Figure 1 represents the dynamics of the skill premium and the relative supply of high-skilled labor (of skills) in the US over the period from 1970 to 2005. It can be seen that during the 1970s, the skill premium was mostly declining, but then the situation reversed and the skill premium started to increase. Additional interest to this pattern is provided by the fact that the relative supply of skills steadily increased over the whole period. Some papers also emphasize significant decline in real wages of different groups of low-skilled workers in the US over the last decades (see Acemoglu and Autor 2011; Krugman and Lawrence 1993).

Figure 1: Skill Premium and Relative Supply of Skills in the U.S. (1970-2005)

Notes: Authors’ calculations based on EU KLEMS Growth and Productivity Accounts

Attempts to explain the determinants and the dynamics of wage differentials between high-skilled and low-skilled labor in the U.S. and European countries (see e.g. Berman et al. 1998; Machin and Van Reenen 1998) have formed one of the most controversial topics in economics. Factor-biased technological change (FBTC) is often considered to be the main factor affecting skill premium. In particular, many influential papers (e.g. Katz and Murphy 1992; Acemoglu 1998, 2002a, 2002b; Caselli and Coleman 2002; Goldin and Katz 2008) emphasize the crucial role of FBTC, namely, skill-biased technological change (SBTC) in reshaping the U.S. labor market. Some recent studies also attempt to separate the impact of this kind of technological improvements from closely related effects, such as the complementarity between capital and high-skilled labor (e.g. McAdam and Willman 2016), as well as to determine the direction of bias of particular technologies (see e.g. Akerman et al. 2015).

In contrast to many comprehensive but “single-country” studies, in this paper we provide a
cross-country analysis of trends in the skill premium and in the relative supply of high-skilled labor based on the data for 21 OECD (US and 20 European) countries over the last decades. We find that, in contrast to the steadily increasing trends in the relative supply of high-skilled labor, the dynamics of the skill premium varies substantially across the countries.

Since the seminal work of Katz and Murphy (1992), a simple model of demand and supply for skills, also known as the canonical model (see Acemoglu and Autor 2011), is treated as the workhorse of theories of wage skill inequality. Technology in this model is assumed to take factor-augmenting form, capturing the influential $i$ that technological change is not necessarily factor-neutral (see the discussion in Acemoglu 2015). This model implies that the rapid growth of the skill premium in the U.S. since 1980’s is associated with the skill-biased technological change. In the present paper we propose estimation of the canonical model based on the so-called steady-demand hypothesis (see Acemoglu 2002b) for the subsample of the initial range of countries. We find the evidence of skilled-biased technological change for all these economies.

Important question concerns also the nature of the bias of technology: What determines whether technological change is skill-biased or, on the contrary, unskill-biased? To cope with this issue, Acemoglu (1998), (2002a), (2009) develops the baseline directed technological change model, where there are two main forces that determine the profitability of developing particular technologies: the price effect and the market size effect. While the former relates to a cheaper production possibility, the latter implies that it is more profitable to develop technologies that would have a (relatively) larger market. Thus, the direction of the technological change is endogenous and depends crucially on the market size for each kind of technologies (i.e. the relative supply of skills) as well as on the elasticity of substitution between skilled and low-skilled labor. In particular, it follows from the model that the elasticity of substitution between production factors is sufficiently large (namely, greater than 2 for the basic version of the model), an increase in the relative supply of a factor decreases its marginal product in the short-run, but induces sufficiently strong technological change biased towards that factor which increases its relative marginal product or, in other words, demand for this factor in the long-run. This effect is also referred to as strong equilibrium (relative) bias of technology (as in Acemoglu 2009) or, in other words, “strong induced-bias hypothesis” (as in Acemoglu 2002a).

Acemoglu’s baseline model is very influential in explaining the decrease in the skill premium in the US during the 1970’s and its significant increase starting around the 1980’s despite the rapid increase in the relative supply of high-skilled labor. It seems that the only shortcoming of the directed technological change model is the necessity of a rather strong assumption on the value of the elasticity of substitution for the presence of strong equilibrium (relative) bias of technology: it

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1Countries with relatively short time period of the data available were excluded from the initial sample of 21 countries.

2Hanlon (2015) provides a historical evidence of the strong induced-bias hypothesis by studying the impact of the U.S. Civil War on the British cotton textile industry.
has to be greater than 2. However, estimating a version of the canonical model, Katz and Murphy (1992) find out that the elasticity of substitution between high-skilled and low-skilled labor is about 1.41. Acemoglu and Autor (2011), using the similar approach with extended data, derive slightly higher value, 1.68. In the present paper, after testing the steady-demand hypothesis for a number of sample countries (see Table 1), we find that in some countries the elasticity of substitution between skilled and low-skilled labor is sufficiently lower than 2, while they experienced both the rapid increase in the skill premium and the relative supply of skills. It motivates us to develop a modification of the baseline directed technological model that requires a modest value of elasticity of substitution for the presence of strong equilibrium (relative) bias of technology. This model is exposed in Section 3.

The rest of the paper is organized as follows. Section 2 provides empirical estimation of the canonical model as well as numerous cross-country comparisons. Section 3 presents the modification of the directed technological change model. Section 4 concludes. Additional results of the empirical estimation are represented by figures provided in Appendices.

2 Empirical Analysis of the Skill Premium

2.1 Data

The main database used in the present research for calculation of the wage premium and the relative high-skilled supply is the EU KLEMS Growth and Productivity Accounts\(^1\). In particular, this data base contains data about shares of labor of three different qualification groups (high, medium, low)\(^2\) in the total wage bill, as well as about a share of each qualification group in the total volume of worked hours for a number of OECD countries for a time period before 2005.

As it has been already mentioned, the skill premium is a wage ratio of high-skilled to low-skilled labor. We take as high-skilled labor the group with the highest qualification among the three groups, and as low-skilled labor – a weighted average of two other qualification groups in the EU KLEMS data. Correspondingly, the skill premium, \(\omega\), is calculated for each country by use of the following formula:

\[
\frac{\varphi_{wh}}{\varphi_h} : \frac{\varphi_{wm} + \varphi_{wl}}{\varphi_m + \varphi_l} = \omega,
\]

where \(\varphi_{wh}, \varphi_{wm}, \varphi_{wl}\) are shares of the high \((h)\), middle \((m)\), and low \((l)\) qualification labor in the total wage bill, while \(\varphi_h, \varphi_m, \varphi_l\) denote corresponding shares in the total volume of worked hours.

The relative supply of the high-skilled labor, in its turn, can be approximately calculated as

\(^1\) URL: http://www.euklems.net/

\(^2\) See O’Mahony and Timmer (2009) for information about skills group formation for this database.
follows

\[
\frac{\varphi_h}{\varphi_m + \varphi_l} = \frac{H}{L}
\]

Figure 2 compares the results of the skill premium from this study with the college/high school wage gap calculated by Autor et al. (2008) for the period 1970 – 2005.

Figure 2: Comparison of the results of the skill premium calculations for the U.S., 1970-2005

It can be easily seen that the results of our calculation of the skill premium strongly correlate with the results obtained by Autor et al. (2008), but our results give higher values of the skill premium, which can be explained by different ways of definition of high-skilled and low-skilled labor and seems rather natural.

2.2 Trends in the Skill Premium and the Relative Supply of Skills: Cross-Country Analysis

In this section, we provide a comparison of dynamics of inequality in payment of labor of different qualification, as well as dynamics of relative supply of skilled labor on base of the sample of 21 OECD countries. The data for each of the countries are presented in Appendix A in form of corresponding graphs. From those country-specific figures one can see that the share of skilled labor increased steady, in one or another degree, absolutely in all 21 countries during the last decade or several decades before 2005. The highest increase of the relative share of skilled labor was observed in Finland and in the US.

Dynamics of the skill premium varies considerably across the countries. In particular, in Austria, Denmark, Finland, France, and Netherlands, the skill premium had a tendency to decline during the whole period of available data before 2005. In opposite, in such countries as the
US, Germany, and Luxembourg, the skill premium, on average, has increased. There are also countries (e.g. Italy) in which the skill premium during the period did not change on average, but considerably fluctuated during the whole period; and, finally, in some countries (Czech Republic, Ireland) the skill premium was on an approximately constant level during the whole period. More detailed results can be seen from country-specific figures in Appendix A. If the skill premium is considered in terms of absolute units, it also considerably varies across countries. Figure 3 provides in visible way results of such comparison.

![Figure 3: The Skill Premium in 1995 and 2005: Cross-Country Data.](image)

Notes: Authors’ calculations based on EU KLEMS Growth and Productivity Accounts.

It can be seen that the value of the skill premium achieved in the US does not stand out compared to other countries in the sample. The highest skill premium is observed in such countries of continental Europe as Portugal, Czech Republic, and Hungary. Remarkably, in all these countries the skill premium increased in 1995-2005.

### 2.3 The Canonical Model of Demand for and Supply of Skills

The canonical model (see e.g. Acemoglu and Autor 2011) serves as a benchmark in contemporary research devoted to studying dynamics of wage skill differentials. In this model two factors (high-skilled and low-skilled labor) produce two imperfectly substitutable intermediate goods, or produce one aggregate good. In particular, let us consider the latter case. In such situation it is common to use the constant elasticity of substitution (CES) production function of the following form:

\[
Y(t) = \left[ a(A_H(t)H(t))^{\frac{\sigma-1}{\sigma}} + b(A_L(t)L(t))^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},
\]

where \(Y(t)\) is output; \(H(t), L(t)\) are quantities of high-skilled and low-skilled labor, correspondingly; \(A_H(t) A_L(t)\), are technological parameters which reflect the measure in which the
technological progress is embodied in one or another factor; \( \sigma \in (0, 1) \cup (1, \infty) \) is the elasticity of substitution between high- and low-skilled labor; \( a, b \) are positive exogenous parameters which reflect a relative “importance” of each of the factors in production.

Note that in the case of the unit elasticity of substitution, \( \sigma = 1 \), and if \( a = \lambda, \; b = (1 - \lambda) \), eq. (2.1) corresponds to the Cobb-Douglas production function

\[
Y(t) = (A_H(t)H(t))^{\lambda} (A_L(t)L(t))^{1-\lambda}.
\]

If the markets for production factors are competitive, then the payoff of each factor is equal to its marginal product. Thus, the skill premium, \( \omega \), is equal to the marginal rate of technical substitution (MRTS) of the factors in the production function. Thus, for the CES production function (2.1), the skill premium in each period of time is

\[
\omega(t) = \left( \frac{a}{b} \right) \left( \frac{A_H(t)}{A_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left( \frac{L(t)}{H(t)} \right)^{\frac{1}{\sigma}}. \tag{2.2}
\]

It follows from eq. (2.1) that the skill premium negatively depends of the relative supply of high-skilled labor under any value of the parameter of the factor elasticity of substitution. The influence of the ratio of the corresponding technological terms, \( A_H(t) \) and \( A_L(t) \), is ambiguous – it depends on the elasticity of substitution. Let us consider the ambiguity in detail.

An increase of the ratio of technological terms, \( A(t) = A_H(t)/A_L(t) \), will be referred as a (relatively) high-skilled labor-augmenting technological change, while a decrease of \( A(t) \) will be referred as a (relatively) low-skilled labor-augmenting technological change. Under a skill-biased technological change (SBTC), we call any (exogenous or endogenous) change of technological parameters leading to increase of the skill premium. Similarly, an unskill-biased technological change (UBTC) is any change of technological parameters leading to decrease of the skill premium. Thus, it can be easily verified that, in case of CES production function (2.1), if \( \sigma > 1 \), then any (relatively) high-skilled labor-augmenting technological change is SBTC, while any (relatively) low-skilled labor-augmenting technological change is UBTC. Conversely, if \( \sigma < 1 \), then any (relatively) low-skilled labor-augmenting technological change is SBTC, while any (relatively) high-skilled labor-augmenting technological change is UBTC (see also Acemoglu 1998, 2002a, 2009). Note also that in the case of the Cobb-Douglas production function (\( \sigma = 1 \)), the technological change of any type is “neutral”, i.e. it does not influence the value of the skill premium.

As it has been already mentioned, according to a number of empirical papers (see e.g. Katz and Murphy 1992; Acemoglu 2002b; Acemoglu and Autor 2011), the elasticity of substitution between high- and low-skilled labor in the US is higher than one and, probably, lies in the interval between 1.4 and 2. In the Section 2.4, we provide an econometric estimation of the canonical model for a sample of OECD countries including the US, which allows us to implicitly estimate the elasticity of substitution between the high- and low-skilled labor in each of these countries.
2.4 The Steady-Demand Hypothesis

Returning to consideration of the canonical model, let us rewrite eq. (2.2) in logarithms:

\[
\ln \omega(t) = \ln \left( \frac{a}{b} \right) + \frac{\sigma - 1}{\sigma} \ln \left( \frac{A_H(t)}{A_L(t)} \right) - \frac{1}{\sigma} \ln \left( \frac{H(t)}{L(t)} \right). \tag{2.3}
\]

To estimate this equation, we exploit the widespread steady-demand hypothesis. It implies that factor-biased technological change takes place with a constant pace over time. Put differently, we assume that

\[
\ln \left( \frac{A_H(t)}{A_L(t)} \right) = \gamma_0 + \gamma_1 t \tag{2.4}
\]

where \( \gamma_1 \) is the difference of growth rates of high- and low-skilled labor. Hence, it is easily verified that (2.3) can be rewritten as follows.

\[
\ln \omega(t) = \text{const} + \frac{\sigma - 1}{\sigma} \gamma_1 t - \frac{1}{\sigma} \ln \left( \frac{H(t)}{L(t)} \right) \tag{2.5}
\]

There are a number of studies focusing on the estimation of similar equation for the U.S. (Katz and Murphy 1992; Acemoglu 2002b; Acemoglu and Autor 2011). We, in turn, apply this hypothesis to the sample of 8 OECD countries. Table 1 represents results of the OLS estimation of this regression equation for the sample.

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Time</th>
<th>H/L Relative Supply</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>1980-2005</td>
<td>0.006 (0.003) ***</td>
<td>-0.357 (0.079) *</td>
<td>0.97</td>
</tr>
<tr>
<td>Denmark</td>
<td>1980-2005</td>
<td>0.011 (0.008)</td>
<td>-0.458 (0.174) ***</td>
<td>0.92</td>
</tr>
<tr>
<td>Finland</td>
<td>1970-2005</td>
<td>0.015 (0.005) ***</td>
<td>-0.576 (0.113) ***</td>
<td>0.85</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1979-2005</td>
<td>0.006 (0.012)</td>
<td>-0.323 (0.250)</td>
<td>0.57</td>
</tr>
<tr>
<td>Spain</td>
<td>1980-2005</td>
<td>0.021 (0.006) ***</td>
<td>-0.583 (0.122) ***</td>
<td>0.86</td>
</tr>
<tr>
<td>Sweden</td>
<td>1981-2005</td>
<td>0.002 (0.001) **</td>
<td>-0.158 (0.026) ***</td>
<td>0.83</td>
</tr>
<tr>
<td>UK</td>
<td>1970-2005</td>
<td>0.003 (0.003)</td>
<td>-0.094 (0.043) **</td>
<td>0.45</td>
</tr>
<tr>
<td>USA</td>
<td>1970-2005</td>
<td>0.020 (0.002) ***</td>
<td>-0.446 (0.063) ***</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses: ***,**,* - significance at the 1, 5 and 10% levels. Authors’ calculations based on EU KLEMS Growth and Productivity Accounts.

It can be easily seen from Table 1 that all countries of the sample experienced high-skilled labor-augmenting technological change. Moreover, SBTC also seems to be observed in all countries of the sample. These implications directly follows from the definitions defined above and the estimates of elasticity of substitution between high-skilled and low-skilled labor, which can be

\(^3\)Countries with relatively short time period of the data available were excluded from the initial sample of 21 countries.
derived from Table 1. Put differently, it follows from (2.3), that the estimates of elasticity of substitution between high-skilled and low-skilled labor can be derived from

\[ \sigma = -\left( \frac{\partial \ln(\omega(t))}{\partial \ln \left( \frac{H(t)}{L(t)} \right)} \right)^{-1}. \]  

(2.6)

According to the estimation results, the value of the elasticity of substitution between high- and low-skilled labor in most of the sample countries is greater than 2. However, for such countries as, e.g., Spain and Finland, the estimated value of the elasticity of substitution is lower than this threshold value, as will be seen from the further analysis (see Section 3). It is also worth mentioning that the OLS estimation of eq. (2.5) potentially faces with the problem of autocorrelation, which is also marked by Acemoglu (2002b). Another serious problem for estimation is a possibility of influence of labor market institutions, trade barriers, and other factors which might change considerably during the period of observation. It is possible that analysis with account for influence of additional factors may lead to slightly different estimates of the elasticity of substitution between high-skilled and low-skilled labor. Appendix B separately represents both the observed and predicted values of the skill premium for each country from Table 1.

3 Directed Technological Change

In this section, we propose a modification of the Acemoglu’s (2002a, 2009) baseline directed technological change model. Allowing for strong equilibrium (relative) bias of technology mentioned above, this model is originally aimed to explain dynamics of the skill premium in the US over the last three decades of twentieth century on base of the market size effect assumption. However, as it will be seen later, our modification provides less restrictive condition for strong (relative) equilibrium bias, which is reasonable for the adoption of directed technological change model to wide cross-country patterns of the skill premium and relative supply of skills.

Let there be high-skilled \( H \) and low-skilled \( L \) labor, both uniformly distributed across (a constant number of) households. The representative household maximizes the utility from consumption on the infinite time interval:

\[ U = \int_{0}^{\infty} e^{-\rho t} \left( \frac{C(t)^{1-\theta} - 1}{1 - \theta} \right) dt, \]

where \( C(t) \) is consumption at moment \( t \); \( \rho \) and \( \theta \) are (positive) exogenous parameters.

The economy produces two intermediate goods and one final good. The production function describing the final production has the CES specification:

\[ Y(t) = \left[ \lambda Y_H(t)^{\frac{1-\theta}{\tau}} + (1 - \lambda)(Y_L(t))^{\frac{1-\theta}{\tau}} \right]^{\frac{\theta}{\tau}}, \]  

(3.1)
Y_H, Y_L - outputs of the two (factor-specific) intermediate goods; \( \varepsilon \in (0, 1) \cup (1, \infty) \) denotes the elasticity of substitution between the intermediate goods.

The final output, \( Y(t) \), is distributed between total consumption, \( C(t) \), expenditures for production of labor-specific machines, \( X(t) \), and R&D expenditures, \( Z(t) \), on invention of new machines, which complement either high-skilled or low-skilled labor, \( Z(t) = Z_H(t) + Z_L(t) \).

The markets of intermediate goods are competitive. For production of each intermediate good, the labor of one specific qualification (correspondingly, high- and low-skilled labor) and a continuum of types of machines (from 0 to \( N_i(t) \), where \( i = H, L \)) are used. The corresponding production functions for the sectors of intermediate goods takes the following form:

\[
Y_H(t) = \frac{1}{1 - \beta} \left( \int_0^{N_H(t)} x_H(\nu, t)^{1-\beta} d\nu \right) \cdot H^\beta; \quad Y_L(t) = \frac{1}{1 - \beta} \left( \int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu \right) \cdot L^\beta, \quad (3.2)
\]

where \( N_H(t) \) and \( N_L(t) \) denote the range of each type of machines available at moment \( t \) for production of corresponding intermediate goods; \( x_i(\nu, t) \) is the quantity of machines of the type \( \nu \in [0, N_i(t)] \) \( i = H, L \) used in production of the corresponding intermediate good, \( 0 < \beta < 1 \) is an exogenous parameter.

It is assumed that the machines fully depreciate in production of the intermediate goods. The machines are supply by monopolists, who, after inventing a variety of machine, possess a perpetual patent for its production and sell it at a price \( p_i^\tau(\nu, t) \) \( i = H, L \). It is assumed that the costs of production of any variety of machines is equal to a fixed value \( \psi = 1 - \beta \) (in final good units). Total expenditures for production of machines are

\[
X(t) = (1 - \beta) \left( \int_0^{N_H(t)} x_H(\nu, t)^{1-\beta} d\nu + \int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu \right),
\]

The present value of the monopolist’s profit is

\[
V_i(\nu, t) = \int_t^\infty \exp \left[ - \int_t^s r(s') ds' \right] \pi_i(\nu, s) ds, \quad i = H, L,
\]

where \( r(t) \) is the interest rate and \( \pi_i(\nu, s) = p_i^\tau(\nu, t) \cdot x_i(\nu, t) - \psi x_i(\nu, t) \) is the monopolist’s profit at time \( t \).

For this model, the skill premium can be expressed as (see also Acemoglu 2002a, 2009):

\[
\omega(t) = \left( \frac{\lambda}{1 - \lambda} \right)^\frac{\varepsilon}{2} \left( \frac{N_H(t)}{N_L(t)} \right)^{\frac{\varepsilon-1}{2}} \left( \frac{L}{H} \right)^{\frac{\varepsilon}{2}}, \quad (3.3)
\]

where \( \sigma \) and \( \varepsilon \) are related as \( \sigma \equiv \varepsilon - (\varepsilon - 1) (1 - \beta) \).

Note that the ratio \( N_H(t)/N_L(t) \) in eq. (3.3) plays a similar role as \( A_H(t)/A_L(t) \) in the canonical
model described above, but improvements in technologies are now endogenous.

As usually, the first order condition of the maximization of the present value of utility implies that in equilibrium the following condition is fulfilled ¹:

\[
\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\theta}.
\]

It is also shown by Acemoglu (2002a),(2009), that on the balanced growth path (BGP) the relative present value of profits of monopolists in the two sectors is

\[
\frac{V_H}{V_L} = \left(\frac{p_H}{p_L}\right)^\frac{\beta}{\beta - 1} H/L,
\]

where \(p_H, p_L\) are prices for intermediate goods, which are constant on the BGP.

It can be seen that eq. (3.4) reflects the abovementioned effects of price and of market size. In particular, incentives for inventing machines to be used by the high-skilled workers increase when the relative price of the corresponding intermediate good increases (the price effect) and/or the relative supply of the high-skilled labor increases (the market size effect).

Now, we introduce an assumption which distinguishes this modification from the baseline version of the model. This assumption relates the form of the innovation possibilities frontier. This “frontier” is a set of equations which describe inventing of new varieties of machines. We suppose the following form of the innovation possibilities frontier:

\[
\dot{N}_H(t) = \eta_H Z_H(t) N_H(t)^{(1+\delta)/2} N_L(t)^{(1-\delta)/2} H^\alpha,
\]

\[
\dot{N}_L(t) = \eta_L Z_L(t) N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} L^\alpha.
\]

In the same way as in the extended Acemoglu’s model (2002a, 2009), (3.5) implies so called state dependence, which is defined by parameter \(\delta\) (0 ≤ \(\delta\) ≤ 1). The sense of it is that the velocity of inventing of new machines may depend positively on the “number” of already existing varieties of machines of the same type, because of the knowledge spillovers. Also the velocity depends negatively on the number of the varieties of machines of the opposite type and positively on the investments to the R&D sector. We also introduce a new assumption which seems to be quite natural: the velocity of inventing of new machines intended for the use by high-skilled labor depends on the number of the high-skilled workers. Similarly, the velocity of inventing of new machines intended for the use by low-skilled labor depends on the number of the low-skilled workers. The degree of this dependency in each case is expressed by a value of parameter \(\alpha\) ≥ 0. Such assumption is motivated by the usual “learning by doing” argument: the experience of the workers (of definite qualification) simplifies the implementation of new varieties.

Assuming a possibility of free entering of new agents into the R&D sector, one can easily

\(^{1}\)It is assumed that the respective transversality condition is satisfied (see e.g. Acemoglu 2009).
verified that on the BGP the following condition is fulfilled,

$$\eta_H N_H(t)^\delta \pi_H H^\alpha = \eta_L N_L(t)^\delta \pi_L L^\alpha. \tag{3.6}$$

Combining (3.6) with the following calculations, it yields the following equilibrium relation for the quantities of machines,

$$\left(\frac{N_H}{N_L}\right)^* = \eta^{\frac{\sigma}{1 - \sigma}} \left(\frac{\lambda}{1 - \lambda}\right)^{\frac{\sigma}{1 - \sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma - 1 + \alpha \sigma}{1 - \sigma}}. \tag{3.7}$$

By use of eqs. (3.3) and (3.7), we obtain the equilibrium skill premium as a function of relative supply of high-skilled labor,

$$\omega^* = \eta^{\frac{\sigma - 1}{1 - \sigma}} \left(\frac{\lambda}{1 - \lambda}\right)^{\frac{(1 - \delta) \sigma}{1 - \sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma - 2 + \delta + \alpha \sigma - \alpha}{1 - \sigma}}. \tag{3.8}$$

Acemoglu (2002a),(2009) introduced a notion of strong equilibrium (relative) bias of technology, which takes place when an increase in the relative supply of a factor induces a sufficiently strong technological change biased towards itself, that leads to an increase in its relative marginal productivity in the long-run. Put differently,

$$\frac{d(\omega^*)}{d(H/L)} > 0 \tag{3.9}$$

Note that (3.8) explicitly establishes conditions for the strong equilibrium (relative) bias in the model. Following Acemoglu (2002a),(2009), we restrict my attention to the cases where $$\sigma < 1/\delta$$ and the stability condition is satisfied. The following proposition formally poses the less restrictive condition on threshold level of elasticity of substitution between skilled and low-skilled labor for strong (relative) equilibrium bias than in the baseline version of the model.

**Proposition 1.** In the modified version of the directed technological change described above, if

$$\sigma > \frac{2 - \delta + \alpha}{1 + \alpha}, \tag{3.10}$$

then there is strong equilibrium (relative) bias of technology.

**Proof.** It is straightforward to derive from (3.8) that (3.9) is equivalent to $$\frac{\sigma - 2 + \delta + \alpha \sigma - \alpha}{1 - \delta \sigma} > 0$$. If $$\sigma < 1/\delta$$ (by initial assumption), then this inequality is equivalent to $$\sigma - 2 + \delta + \alpha (\sigma - 1) > 0$$. After rearranging terms, it yields (3.10). Q.E.D.

It is easily verified that the new condition (3.10) is less restrictive (in case of $$\alpha > 0$$) than the equality of the threshold level to 2 (in the baseline version of the model) or to 2 − $$\delta$$ (in the extended version proposed by Acemoglu 2002a, 2009). Put differently, 1 ≤ $$\frac{2 - \delta + \alpha}{1 + \alpha}$$ ≤ 2 − $$\delta$$ ≤ 2. Moreover,
it can be easily shown that the threshold level for strong (equilibrium) relative bias depends negatively from \( \alpha \): 

\[
\frac{\partial \left( \frac{2-\delta+\alpha}{2+\alpha} \right)}{\partial \alpha} = \frac{\delta-1}{(1+\alpha)^2} < 0.
\]

Figure 4 represents this relationship graphically (for simplicity, it is assumed that \( \delta = 0 \)).

Figure 4: Dependence of the threshold level of elasticity of substitution (EoS) on \( \alpha \).

Returning to the results of empirical analysis provided in Section 2.4, it can be concluded that the modification of the directed technological model can be applied to explanation of wider cross-country patterns of the skill premium and relative supply of skills. Put differently, the newly modified directed technological change model can be useful in explaining the dynamics of skill premium in countries (e.g. Finland and Spain) with estimated elasticity of substitution between skilled and low-skilled labor lower than 2, but higher than 1.

4 Conclusion

This paper is devoted to studying the factors affecting the dynamics of the skill premium in a number of European countries as well as in the US. In particular, we provide the cross-country analysis of trends in the skill premium and in the relative supply of high-skilled labor based on the data for 21 OECD (US and 20 European) countries over the period. We document that, in contrast to the steadily increasing trends in the relative supply of high-skilled labor, the dynamics of the skill premium varies substantially across the countries, but these trends do not diverge very much.

While it is a broad consensus that two main factors affecting wage inequality are factor-biased (namely, skill-biased) technological change and international trade, this paper has been mainly focused on the impact of factor-biased technological change. In particular, adapting a methodology first pioneered by Katz and Murphy (1992), we find the evidence that most all countries for which we have sufficiently long period of observations experienced skill-biased technological change over
the sample period. The other finding is that the estimated elasticity of substitution between skilled and low-skilled labor in some countries (e.g. Finland and Spain) is higher, than 1, but significantly lower, than 2. At the same time, the baseline directed technological change model (Acemoglu 2002a, 2009) implies that the relative (equilibrium) bias of technology holds only when the elasticity of substitution is rather high. All this has been motivated us to develop the modification of the baseline directed technological change model with less restrictive condition on the threshold level of the elasticity of substitution for relative (equilibrium) bias of technology. It has been shown that such modification is helpful in explaining the skill premium patterns for additional range of countries.
References


A Skill Premium and the Relative Supply of Skills: Cross-Country Data

Fig. A1-A20. Skill premium and the relative supply of skills for the sample of OECD countries. Authors’ calculation based on EU KLEMS Growth and Productivity Accounts.
B Observed vs. Predicted Skill Premium (the Steady-Demand Hypothesis)

Fig. B1-B8. Observed vs. predicted (from eq. 2.5) skill premium for 8 OECD countries. Authors’ calculation based on EU KLEMS Growth and Productivity Accounts.